Represent Proportional Relationships

Getting the Idea

A directly proportional relationship has an equation of the form $y = kx$. It is a relationship between two quantities in which one is a constant multiple of the other. When one quantity changes, the other quantity changes by a constant factor, k. The constant factor k is the **constant of proportionality**.

Example 1

The function table below shows the relationship between the side lengths of a regular octagon and its perimeter.

Side Lengths, s (inches)	Perimeter, P (inches)
1	8
2	16
3	24
4	32
9	?

If a regular octagon has side lengths of 9 inches, what is its perimeter?

Strategy   **Write and solve an equation.**

Step 1   Write an equation to represent the situation.

The perimeter is always 8 times the side length of a regular octagon.

So, 8 is the constant of proportionality.

$P = 8s$

Step 2   Substitute the side length of 9 for s and find the perimeter.

$P = 8 \times 9$ in.

$= 72$ in.

Solution   **A regular octagon with a side length of 9 inches has a perimeter of 72 inches.**

A directly proportional relationship is a **linear relationship** because it forms a straight line when graphed. The graph of a proportional relationship is a straight line that passes through the **origin** at (0, 0). It will also pass through the point (1, k), where k is the constant of proportionality, or the unit rate. In a graph of a proportional relationship, the unit rate, or the constant of proportionality, is shown by the **slope**, which measures how steep the line is.

Example 2

An empty swimming pool is being filled at a rate of 10 gallons per minute. Make a graph to display the amount of water in the pool each minute for 6 minutes.

Strategy   **Write an equation and create a function table to represent the situation.**

Step 1   Write an equation to represent the situation.

Let x = the number of minutes and y = the number of gallons.

$y = 10x$

Step 2   Make a function table to show the number of gallons in the pool each minute.

At 0 minutes, when $x = 0$, there is no water going into the pool, so $y = 0$.

At 1 minute, when $x = 1$, the pool is filled with 10 gallons of water.

At 2 minutes, when $x = 2$, the pool is filled with 20 gallons of water.

Complete the rest of the table.

Number of Minutes (x)	0	1	2	3	4	5	6
Number of Gallons (y)	0	10	20	30	40	50	60

**Step 3**   Make a line graph, using the ordered pairs from the function table.

Notice that the graph passes through the origin and the point (1, 10). Remember that 10 is the constant of proportionality, or the unit rate, of 10 gallons per minute.

Amount of Water in Pool
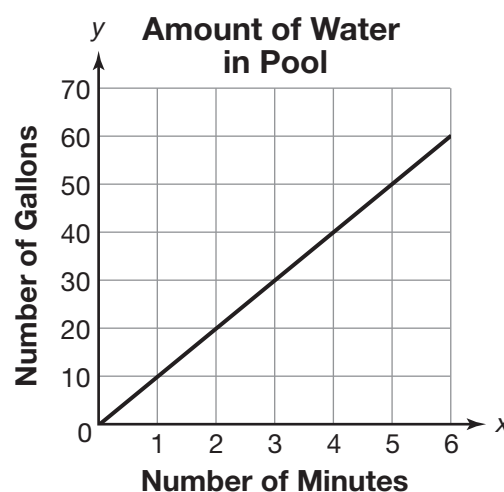
Solution   The graph is shown in Step 3.

Example 3

The graph below shows the amounts charged for purchasing different numbers of roses from a florist. Is there a proportional relationship between the number of roses bought and the cost? If so, what is the constant of proportionality and what does it mean in this context?

Costs of Roses
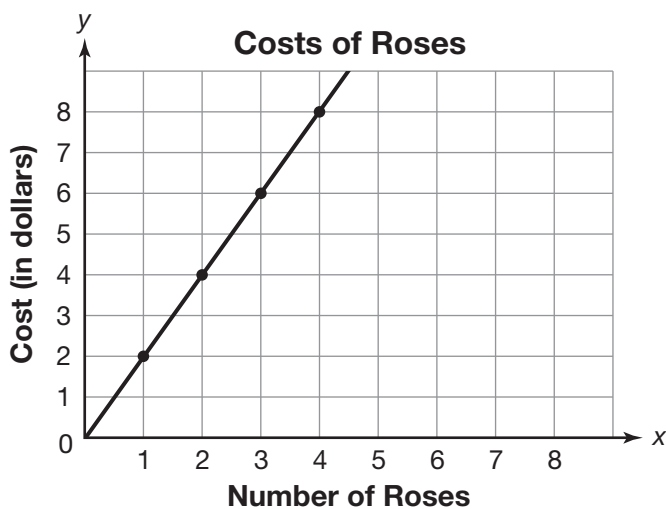
Strategy	**Analyze the graph to determine if the relationship is proportional.**

Step 1 Think about the graph of a proportional relationship.

The graph of a proportional relationship is a straight line that passes through the origin. Since this graph matches that description, it shows a proportional relationship.

Step 2 Determine the constant of proportionality.

The graph must pass through the point $(1, k)$, where k is the constant of proportionality.

The graph passes through the point $(1, 2)$. So, k is 2.

Step 3 Determine what the constant of proportionality means in this context.

In this situation, the fact that $k = 2$ means that each rose costs $2.

Solution **The relationship is proportional and the constant of proportionality, 2, means that each rose costs $2.**

Coached Example

A movie theater charges $8 per movie ticket. How much would it cost for six people? Make a graph to represent the situation.
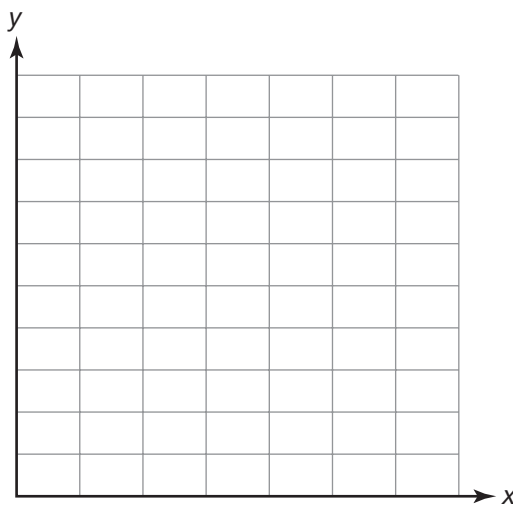
Let x represent the number of tickets.

Let y represent the cost, in dollars.

Write an equation to represent the situation. _____

Complete the function table.

Number of Tickets (x)	0	1	2	3	4	5	6
Cost in Dollars, (y)							

Create a graph to show the values in the table.

The graph passes through the point (1, _____). So, _____ is the constant of proportionality, or the unit rate.

It would cost _____ for six people.

Lesson Practice

Choose the correct answer.

Use the function table for questions 1 and 2.

The table shows the relationship between the side lengths of a regular pentagon and its perimeter.

Side Length, s (inches)	Perimeter, P (inches)
1	5
2	10
3	15
4	20
5	25

1. Which equation shows the relationship between the side length and the perimeter of a regular pentagon?

   A. $P = s + 5$

   B. $P = 5s$

   C. $P = \frac{1}{5}s$

   D. $P = 5s + 5$

2. If a regular pentagon has side lengths of 8 inches, what is its perimeter?

   A. 13 inches

   B. 30 inches

   C. 40 inches

   D. 45 inches

Use the graph for questions 3–5.

The graph shows the relationship between the cost and the number of uniforms ordered by a sports team.
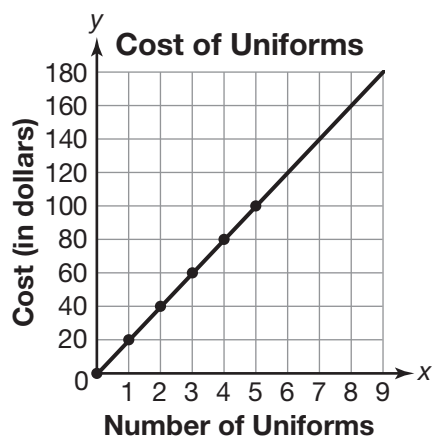
3. Which equation shows the relationship between the number of uniforms, x, and the cost, y?

   A. $y = 20x$ **C.** $y = 2x$

   B. $y = 10x$ **D.** $y = x$

4. What is the unit cost of a uniform?

   A. $2 per uniform

   B. $5 per uniform

   C. $10 per uniform

   D. $20 per uniform

5. How much will it cost the team to order 8 uniforms?

   A. $20   **C.** $160

   B. $140   **D.** $180

6. Amber rides 30 miles in 2 hours. Which equation shows the relationship between the distance, d, and the time, t, that she rides?

A. $d = 2t$

B. $d = 15t$
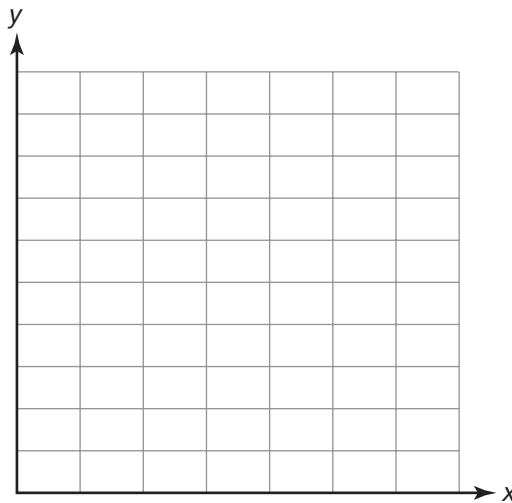
C. $d = 30t$

D. $d = 32t$

7. Pasha bought 3 pounds of onions for $2.67. Which ratio is proportional to 3 pounds at $2.67?

A. $\dfrac{\$3.48}{4 \text{ pounds}}$

B. $\dfrac{\$3.67}{4 \text{ pounds}}$

C. $\dfrac{\$4.45}{5 \text{ pounds}}$

D. $\dfrac{\$4.57}{5 \text{ pounds}}$

8. River Rambler charges $25 per day to rent a kayak. How much will it cost to rent a kayak for 5 days?

A. Write and solve an equation to solve the problem.

_____

_____

_____

B. Make a graph to display the relationship.

9. Look at each equation. Does the equation represent a directly proportional relationship?
Select Yes or No.

A. $y = 4x + 1$ ○ Yes   ○ No

B. $y = 3x - 1$ ○ Yes   ○ No

C. $\frac{y}{x} = 5$ ○ Yes   ○ No

D. $y = x$ ○ Yes   ○ No

E. $\frac{x}{y} = 7$ ○ Yes   ○ No

F. $xy = 8$ ○ Yes   ○ No

10. Which table represents a directly proportional relationship? Circle all that apply.

A.

x	1	2	3	4
y	4	8	12	16

B.

x	1	2	3	4
y	9	8	6	5

C.

x	1	2	3	4
y	5	10	15	20

D.

x	1	2	3	4
y	2.5	6	7.5	12

E.

x	1	2	3	4
y	0.5	1	1.5	2

11. Is each ratio of cost per pound proportional to $23.40 for 4 pounds of trail mix?
Select Yes or No.

A. 1 pound for $5.85   ○ Yes   ○ No

B. 2 pounds for $11.75   ○ Yes   ○ No

C. 5 pounds for $29.25   ○ Yes   ○ No

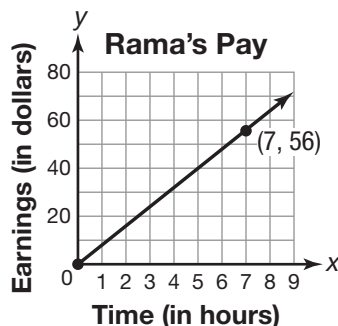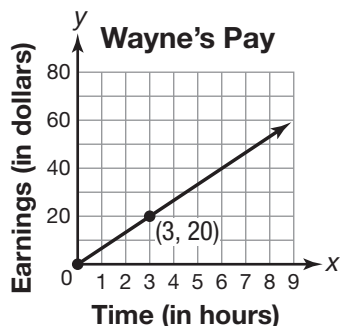D. 7 pounds for $45.20   ○ Yes   ○ No

E. 12 pounds for $70.12   ○ Yes   ○ No

12. The table shows distances that a train travels while going at a constant speed. Use numbers from the box to make the table represent a directly proportional relationship.

Time (h), x	0	3		6	9	
Distance (mi), y	0		360	432		

4	216
5	648
11	791
12	864

13. The graphs show Wayne's and Rama's pay for helping their parents do yard work. Write each point in the correct box.

Wayne's Pay — Earnings (in dollars) vs Time (in hours); point (3, 20)

Rama's Pay — Earnings (in dollars) vs Time (in hours); point (7, 56)

| (8, 52) | (5, 32.5) | (8, 64) | (7, 45.5) | (4, 32) | (5, 40) |

Points on Wayne's Graph	Points on Rama's Graph